

## Factors That Influence High Scores in Higher Secondary Examination - A Nano Topological Approach

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### ABSTRACT

Topology is one of the great unifying ideas of mathematics. In this article, nano topological approach is made use of to reduce attributes in order to analyze the factors influencing high scores in higher secondary examination.

### I. INTRODUCTION

In the present scenario high scores in the Higher Secondary Examinations have become the high order priority in the academic life of a student, as it is the main factor which decides the student's career. Parents have more concern, on their words and they wish them to join a course, particularly a professional course, in a reputed institution. Many students perform upto the expectations. Application of mathematical concepts facilitates the authorities concerned to analyze the issues scientifically and arrive at the most reliable decision. Here we analyze the factors contributing high scores in higher secondary examination. By collecting the real time data from students of 2014 passed out batch and use nano topology to identify the key factor that influence the student's score.

Lellis Thivagar [1] introduced a nano topological space with respect to a subset  $X$  of an universe which is defined in terms of lower and upper approximations of  $X$ . The elements of a nano topological space are called the nano - open sets. He has also studied nano closure and nano interior of a set. Lellis Thivagar [2] has done Nutrition Modelling Through Nano Topology. In this paper, he has applied set - valued ordered information systems in attribute reduction using the basis of nano topology in two real life situations.

### II. PRELIMINARIES

#### Definition 2.1 [2]:

Let  $U$  be a nonempty finite set of objects called the universe and  $R$  be an equivalence relation on  $U$ . Let  $(U,R)$  is said to be an approximation space. Let  $X \subseteq U$ .

#### Definition 2.2 [2]:

Let  $(U,R)$  is said to be an approximation space. Let  $X \subseteq U$ .

The lower approximation of  $X$  with respect to  $R$  is the set of all objects, which can be for certain classified as  $X$  with respect to  $R$  and it is denoted by  $L_R(X)$ . That is  $L_R(X) = \bigcup \{ R(x) \subseteq X \}$ , where  $R(x)$  denotes the equivalence class determined by  $X$ .

#### Definition 2.3 [2]:

The upper approximation of  $X$  with respect to  $R$  is the set of all objects, which can be possibly classified as  $X$  with respect to  $R$  and is denoted by  $U_R(X)$ .

That is  $U_R(X) = \bigcup \{ R(x) \cap X \neq \emptyset \}$ .

#### Definition 2.4 [2]:

The boundary region of  $X$  with respect to  $R$  is the set of all objects which can be classified neither as  $X$  nor as not  $X$  with respect to  $R$  and is denoted by  $B_R(X)$ .

That is  $B_R(X) = U_R(X) - L_R(X)$ .

#### Definition 2.5 [2]:

Let  $U$  be the universe,  $R$  be the equivalence relation on  $U$  and

$\tau_R(X) = \{ U, \emptyset, U_R(X), L_R(X), B_R(X) \}$  where  $X \subseteq U$ .

1.  $U$  and  $\emptyset \in \tau_R(X)$
2. The union of the elements of any sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$
3. The intersection of the elements of any finite sub collection of  $\tau_R(X)$  is in

$\tau_R(X)$

That is  $\tau_R(X)$  is a topology on  $U$  called the nano topology on  $U$  with respect to  $X$ . We call  $(U, \tau_R(X))$  as the nano topological space.

The elements of  $\tau_R(X)$  as nano open sets.

**Definition 2.6 [2] :**

If  $\tau_R(X)$  is the nano topology on U with respect to X then the set  $\beta_R(X) = \{U, L_R(X), B_R(X)\}$  is the basis for  $\tau_R(X)$ .

Two classes are said to be **maximum tolerant** to each other if their attribute values are the same.

**ALGORITHM :**

**Step 1:** Find the maximum tolerance classes of U corresponding to C, Lower and Upper approximation, Boundary region, Nano topology and its basis from the Case 1.

**Step 2:** Remove an attribute x from C and find lower and upper approximations and the boundary region of X corresponding to  $C - \{x\}$ , Generate the nano topology

$\tau_{C-\{x\}}(X)$  on U and its basis  $\beta_{C-\{x\}}(X)$ .

**Step 3:** Set  $M = \{x \in C / \beta_{C-\{x\}}(X) \neq \beta_C(X)\}$

**Step 4:** Repeat steps 2 and 3 for all attributes in C.

**Step 5:** Those attributes in C for which  $\beta_{C-\{x\}}(X) \neq \beta_C(X)$  form the core

**Step 6:** Find the maximal tolerance classes lower, upper approximations, boundary region, Nano topology and its Basis from the Case 2.

**Step 7:** Repeat Step 1 to 5

**Step 8:** Core value from Steps 5 and 7.

**III. HIGH SCORES IN HIGHER SECONDARY EXAM**

$U = \{ B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9, B_{10}, B_{11}, B_{12}, B_{13}, B_{14}, B_{15}, B_{16}, B_{17}, B_{18} \}$

$A = \{ E \text{ (Exposure to scoring techniques)}, B \text{ (Better motivation)}, P \text{ (Proper guidance and counselling)}, L \text{ (Planned Preparation)}, S \text{ (Special coaching imparted by tuition centres)} \}$

$C = \{ E, B, P, L, S \}$

Here U is the universe, A is the set of attributes namely Condition attributes (C) and Decision attributes (D)

To analyze the factors influencing the higher academic performance of higher secondary students opinion of eighteen students from different category was collected. A study was conducted to determine the factors influencing high scores in higher secondary examinations.

E - Exposure to scoring techniques

B - Better motivation

P - Proper guidance and counselling

L - Planned Preparation

S - Special coaching imparted by tuition centres

Each student was asked to give their opinion for each factor. Using the data, a tabular column is framed and the calculation are done.

Here R, instead of equivalence class it is the maximum tolerance class.

PUPIL	E	B	P	L	S	DECISION
B <sub>1</sub>	NO	YES	•	YES	YES	NO
B <sub>2</sub>	YES	•	NO	YES	•	YES
B <sub>3</sub>	YES	NO	•	NO	•	NO
B <sub>4</sub>	YES	YES	YES	•	•	YES
B <sub>5</sub>	NO	YES	NO	•	YES	NO
B <sub>6</sub>	YES	YES	•	YES	NO	YES
B <sub>7</sub>	•	YES	•	YES	YES	NO
B <sub>8</sub>	NO	YES	•	•	YES	NO
B <sub>9</sub>	YES	•	•	YES	•	YES
B <sub>10</sub>	YES	NO	NO	YES	•	YES
B <sub>11</sub>	YES	•	•	NO	•	NO
B <sub>12</sub>	YES	NO	NO	NO	NO	NO
B <sub>13</sub>	•	YES	YES	•	•	YES
B <sub>14</sub>	YES	YES	YES	•	NO	YES
B <sub>15</sub>	NO	NO	NO	•	YES	NO
B <sub>16</sub>	NO	•	NO	•	YES	NO
B <sub>17</sub>	YES	YES	•	YES	•	YES
B <sub>18</sub>	YES	YES	•	NO	NO	YES

• - This symbol denotes that the decision can be either yes or no.

**CASE I: { HIGH PERFORMERS }**

Let X be the set of high performers.

That is,  $X = \{ B_2, B_4, B_6, B_9, B_{10}, B_{13}, B_{14}, B_{17}, B_{18} \}$

$U/R(C) = \{ \{ B_1, B_5, B_7, B_8, B_{16} \} \{ B_1, B_7, B_8, B_{13} \} \{ B_4, B_6, B_9, B_{13}, B_{14}, B_{17} \} \{ B_4, B_7 \} \{ B_4, B_{11}, B_{18} \} \{ B_2, B_6, B_9, B_{17} \} \{ B_2, B_7 \} \{ B_2, B_{10} \} \{ B_6, B_{13}, B_{14} \} \{ B_3, B_{11}, B_{12} \} \}$

Then the lower and upper approximations of X corresponding to C are given by

$$L_C(X) = \{ B_2, B_4, B_6, B_9, B_{10}, B_{13}, B_{14}, B_{17} \}$$

$$U_C(X) = \{ B_1, B_2, B_4, B_6, B_7, B_8, B_9, B_{10}, B_{11}, B_{13}, B_{14}, B_{17}, B_{18} \}$$

$$B_C(X) = \{ B_1, B_7, B_8, B_{11}, B_{18} \}$$

Then the nano topology on U is given by

$$\tau_C(X) = \{ U, \phi, U_C(X), L_C(X), B_C(X) \} \text{ and its base is given by } \beta_C(X) = \{ U, L_C(X), B_C(X) \}$$

**The problem is to find key attributes i.e., factors influencing higher academic performance of higher secondary students.**

### Step 1:

When the attribute "EXPOSURE TO SCORING TECHNIQUES" is removed from C, the lower and upper approximations are given by

$$L_{C-E}(X) = \{ B_2, B_4, B_6, B_9, B_{13}, B_{14}, B_{17} \}$$

$$U_{C-E}(X) = \{ B_1, B_2, B_4, B_5, B_7, B_8, B_9, B_{10}, B_{11}, B_{13}, B_{14}, B_{15}, B_{16}, B_{17}, B_{18} \}$$

The corresponding boundary region is

$$B_{C-E}(X) = \{ B_1, B_5, B_7, B_8, B_{10}, B_{11}, B_{15}, B_{16}, B_{18} \}$$

Therefore, the corresponding nano topology and its basis are given by

$$\tau_{C-E}(X) = \{ U, \phi, U_{C-E}(X), L_{C-E}(X), B_{C-E}(X) \} \text{ and } \beta_{C-E}(X) \neq \beta_C(X)$$

When the attribute "BETTER MOTIVATION" is omitted from C,

The boundary region,  $B_{C-B}(X) = \{ B_1, B_3, B_5, B_7, B_8, B_{11}, B_{18} \}$  and its basis is

$$\beta_{C-B}(X) \neq \beta_C(X)$$

When the attribute "PROPER GUIDANCE AND COUNSELING" is omitted from C,  $B_{C-P}(X) = \{ B_1, B_7, B_8, B_{11} \}$  with the basis  $\beta_{C-P}(X) = \beta_C(X)$

When the attribute "PLANNED PREPARATION" is omitted from C, the base is  $\beta_{C-L}(X) \neq \beta_C(X)$

When the attribute "SPECIAL COACHING IMPARTED BY TUITION CENTRES" is omitted from C,

$$\beta_{C-S}(X) = \beta_C(X)$$

Since  $\beta_{C-S}(X) = \beta_C(X) = \beta_{C-P}(X)$

$C - P = \{ E, B, L, S \}$  and  $C - S = \{ E, B, L, P \}$  are the two reducts. But our problem is to find the **minimal reduct** which is given by the core and which corresponds to the key factors that influence high scores in higher secondary.

### Step 2:

$$\text{Let } K = C - P = \{ E, B, L, S \}$$

$$\text{then } \beta_K(X) = \beta_C(X)$$

Consider  $U/R(K-E) = \{ \{ B_2, B_4, B_6, B_9, B_{13}, B_{14}, B_{17} \} \{ B_2, B_4, B_7, B_8, B_{16} \} \{ B_2, B_{10}, B_{15} \} \{ B_4, B_{11}, B_{18} \} \{ B_2, B_4, B_{14} \} \{ B_4, B_{18} \} \{ B_4, B_5, B_{11} \} \{ B_1, B_2, B_4, B_5, B_7, B_8, B_9, B_{13}, B_{16}, B_{17} \} \{ B_3, B_{11}, B_{12}, B_{15}, B_{16} \} \}$

$$L_{K-E}(X) = \{ B_2, B_4, B_6, B_9, B_{13}, B_{14}, B_{17}, B_{18} \}$$

$$U_{K-E}(X) = \{ B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9, B_{10}, B_{11}, B_{12}, B_{13}, B_{14}, B_{15}, B_{16}, B_{17}, B_{18} \}$$

Hence the base of the corresponding nano topology is  $\beta_C(X) \neq \beta_{K-E}(X)$

When "BETTER MOTIVATION" is removed from K, approximations, boundary region of X and the base of the nano topology corresponding to  $K - B$  are given by

$$L_{K-B}(X) = \{ B_2, B_4, B_6, B_9, B_{10}, B_{13}, B_{14}, B_{17} \}$$

$$U_{K-B}(X) = \{ B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9, B_{10}, B_{11}, B_{12}, B_{13}, B_{14}, B_{15}, B_{16}, B_{17}, B_{18} \}$$

$$B_{K-B}(X) = \{ B_1, B_3, B_5, B_7, B_8, B_{11}, B_{12}, B_{15}, B_{16}, B_{18} \} \text{ and } \beta_{K-B}(X) \neq \beta_C(X)$$

When "PLANNED PREPARATION" and "SPECIAL COACHING IMPARTED BY TUITION CENTRES" are removed from K, the base of the nano topology are given by

$$\beta_{K-L}(X) \neq \beta_C(X), \beta_{K-S}(X) = \beta_C(X)$$

### Step 3:

Let  $Q = K - S = \{ E, B, L \}$  then

$$\beta_Q(X) = \beta_C(X)$$

$$U/R(Q-E) = \{ \{ B_4, B_5, B_{18} \} \{ B_1, B_2, B_4, B_5, B_6, B_7, B_8, B_9, B_{13}, B_{14}, B_{16}, B_{17} \} \{ B_2, B_{10} \} \{ B_3, B_{11}, B_{12}, B_{15}, B_{16} \} \}$$

$$\beta_{Q-E}(X) \neq \beta_C(X)$$

$$\text{Also, } U/R(Q-B) = \{ \{ B_2, B_4, B_6, B_7, B_8, B_9, B_{13}, B_{14}, B_{16}, B_{17} \} \{ B_4, B_{18} \} \{ B_3, B_4, B_{11}, B_{12}, B_{13}, B_{14}, B_{18} \} \{ B_4, B_9, B_{10}, B_{17} \} \{ B_1, B_5, B_7, B_8, B_{13}, B_{16} \} \}$$

$$\beta_{Q-B}(X) \neq \beta_C(X) \text{ and}$$

$$\beta_{Q-L}(X) \neq \beta_C(X)$$

Therefore  $Q = \{ E, B, L \}$  is a **minimal reduct**.

### Step 4:

When  $R = C - S = \{ E, B, P, L \}$ ,

$$\beta_R(X) = \beta_C(X)$$

$$\text{But, } \beta_{R-E}(X) \neq \beta_C(X),$$

$$\beta_{R-B}(X) \neq \beta_C(X), \beta_{R-P}(X) = \beta_C(X),$$

$$\beta_{R-L}(X) \neq \beta_C(X)$$

Thus if  $Z = R - P = \{ E, B, L \}$  we can show that

$$\beta_{Z-\{x\}} \neq \beta_C(X) \text{ for all } x \in Z.$$

Therefore  $Z = \{ E, B, L \}$  is a **minimal reduct**.

**Thus, Core = \{ E, B, L \}**

### Case 2: (STUDENTS WITH LEAST SCORE)

Let X be the set of students with least score.

$$\text{Let } X = \{ B_1, B_3, B_5, B_7, B_8, B_{11}, B_{12}, B_{15}, B_{16} \}$$

The corresponding family of tolerance class is given by

$$U/R(C) = \{ \{ B_1, B_5, B_7, B_8, B_{16} \} \{ B_3, B_{11}, B_{12} \} \}$$

$$\text{Then } L_C(X) = \{ B_1, B_3, B_5, B_7, B_8, B_{11}, B_{12}, B_{16} \} = U_C(X)$$

$$\text{And hence } B_C(X) = \phi$$

Therefore,  $\beta_C(X) = \{U, L_C(X)\}$

When we remove the attributes in C we get the following

$U/R(C-E) = \{\{B_1, B_4, B_7, B_8, B_9, B_{13}, B_{17}\}\{B_1, B_{16}\}\{B_4, B_{14}, B_{18}\}\{B_1, B_2, B_7, B_8, B_9, B_{17}\}\{B_1, B_{13}\}\{B_2, B_{10}, B_{16}\}\{B_3, B_{11}, B_{15}, B_{16}\}\{B_3, B_{11}, B_{12}\}\{B_2, B_5, B_7, B_8, B_9, B_{16}, B_{17}\}\{B_2, B_{10}, B_{15}\}\{B_5, B_{11}\}\}$

$B_{C-E}(X) = \{B_2, B_4, B_7, B_8, B_9, B_{13}, B_{17}\}$  and hence  $\beta_{C-E}(X) \neq \beta_C(X)$

Also,  $U/R(C-B) = \{\{B_1, B_5, B_7, B_8, B_{15}, B_{16}\}\{B_1, B_{13}\}\{B_2, B_6, B_9, B_{17}\}\{B_2, B_7}\{B_6, B_{13}, B_{14}\}\{B_3, B_4, B_{11}, B_{13}, B_{14}, B_{18}\}\{B_3, B_{12}\}\{B_4, B_7, B_9, B_{17}\}\}$

$B_{C-B}(X) = \{B_2, B_4, B_9, B_{11}, B_{13}, B_{14}, B_{17}, B_{18}\}$

Therefore,  $\beta_{C-B}(X) \neq \beta_C(X)$

$U/R(C-P) = \{\{B_1, B_5, B_7, B_8, B_{16}\}\{B_1, B_{13}\}\{B_3, B_{11}, B_{12}\}\}$

$\beta_{C-P}(X) = \beta_C(X), \beta_{C-L}(X) \neq \beta_C(X)$

$U/R(C-S) = \{\{B_1, B_5, B_7, B_8, B_{16}\}\{B_1, B_{13}\}\{B_2, B_6, B_7, B_9, B_{17}\}\{B_2, B_{10}\}\{B_6, B_{13}, B_{14}\}\{B_3, B_{11}, B_{12}\}\{B_4, B_6, B_7, B_9, B_{13}, B_{14}, B_{17}\}\{B_4, B_{11}, B_{18}\}\}$

$\beta_{C-S}(X) = \beta_C(X)$

Thus  $C-P = \{E, B, L, S\}$  and  $C-S = \{E, B, L, P\}$  are the reducts.

As in the previous case, it can be shown that  $C-S \cap C-P = \{E, B, L\} = \text{CORE}$ .

#### IV. OBSERVATION

Since from the two cases,  $\text{CORE} = \{E, B, L\}$ . We conclude that “EXPOSURE TO SCORING TECHNIQUES”, “BETTER MOTIVATION” and “PLANNED PREPARATION” are the key factors that influence higher scores in higher secondary examination.

#### V. CONCLUSION

In this paper, attribute reduction is done using the basis of nano topology in real life situation. Here we show by means of topological reduction that “EXPOSURE TO SCORING TECHNIQUES”, “BETTER MOTIVATION” and “PLANNED PREPARATION” are the key factors that influence higher scores in higher secondary examination. Thus, the basis of nano topology can be applied in many real life situations.

#### REFERENCES:

- [1] M.Lellis Thivagar, Carmel Richard, Note on nano topological spaces (communicated)
- [2] M.Lellis Thivagar, Carmel Richard, Nutrition Modelling Through Nano Topology, Int. Journal of Engineering Research and Applications, Vol 4, Issue 1 (Version 1), (2014), 327-334.

- [3] Z. Pawlak, Rough sets, International Journal of Computer and Information Sciences 11 (1982), 341-356.
- [4] Z. Pawlak, A. Skowron, Rough sets: some extensions, Information Sciences 177 (2006), 28-40.
- [5] Z. Pawlak, A. Skowron, Rudiments of rough sets, Information Sciences 177 (2007), 3-27.